Generalized Discrete Vortex Method for Cylinders Without Sharp Edges

P. A. Smith* and P. K. Stansby†
University of Manchester, Manchester, England

Abstract

VISCOUS flow around a circular cylinder may be simulated by the fractional step method of Chorin, where the inviscid part of the calculation is solved by convecting discrete vortices in a Lagrangian vortex-in-cell scheme with a radially expanding polar mesh and by superimposing random walks to simulate diffusion. This approach is extended to allow nearly arbitrary cylinder shapes, without sharp edges, by a conformal transformation of the shape to a circle. The attractions of the Lagrangian vortex method concerning numerical diffusion, stability, and efficiency are thus transferred to more general shapes, while maintaining high accuracy where it is required. Forces, pressures, and vortex shedding frequencies for eight different shapes were found to be in reasonable agreement with experiments at subcritical Reynolds numbers.

Contents

Flows around several cylinders without sharp edges have been computed by the Lagrangian discrete-vortex scheme first proposed by Chorin for the solution of the vorticity equation.[1] At each time step, a vortex sheet is created along the cylinder surface in order to satisfy the boundary condition of zero tangential velocity. The sheet is then discretized into new vortices, which are added to those representing the vorticity field of the flow. The process of vorticity diffusion is simulated by adding a random perturbation to the position of each vortex. Finally the vortices are convected in the velocity field. Good flow simulations require a very large number of vortices, and their convection may be handled efficiently by the vortex-in-cell method.[2]

The computation is performed in a transformed complex plane ζ. The Theodorsen and Garrick transformation $[z=f(\zeta)]$ is used to map the region outside the cylinder in the physical complex plane z onto the region outside the unit circle $|\zeta|=1$ in the transformed plane. The function f contains a set of mapping coefficients, which are determined for each cylinder by the fast-Fourier-transform method of Ives.[3,4] The vorticity field is modeled, in the physical plane, by a distribution of vortices outside the cylinder and, in the transformed plane, by a distribution of vortices of equal circulations at corresponding points outside the unit circle. $\delta\zeta_D$, the random perturbation simulating diffusion, is given by

$$\delta\zeta_D = \frac{\mathrm{d}\zeta}{\mathrm{d}z}(\sigma_1 + i\sigma_2) \tag{1}$$

where σ_1 and σ_2 are selected at random for each vortex and at each time step from a Gaussian distribution with zero mean and variance $2\upsilon\Delta t$; υ is the kinematic viscosity and Δt is the time step. The vortex-in-cell calculation employs an exponentially expanding polar mesh, defined over an annular domain external to the unit circle. This gives fine definition in the boundary layer where it is required and coarse definition in the far field. The complex velocity of each vortex w is calculated; the vortices are then displaced by an amount $\delta\zeta_C$, given by

$$\delta\zeta_C = \frac{\mathrm{d}\zeta}{\mathrm{d}z}\left(\overline{w\frac{\mathrm{d}\zeta}{\mathrm{d}z}}\right)\Delta t \tag{2}$$

Following this displacement, a new set of vortices is created by a discretization of the sheet of surface vorticity that satisfies the zero tangential velocity boundary condition around the unit circle. The change in pressure (ΔP) across a small element of the surface is proportional to the circulation of the vortex generated by the element Γ.

$$\Delta P = -\rho\frac{\Gamma}{\Delta t} \tag{3}$$

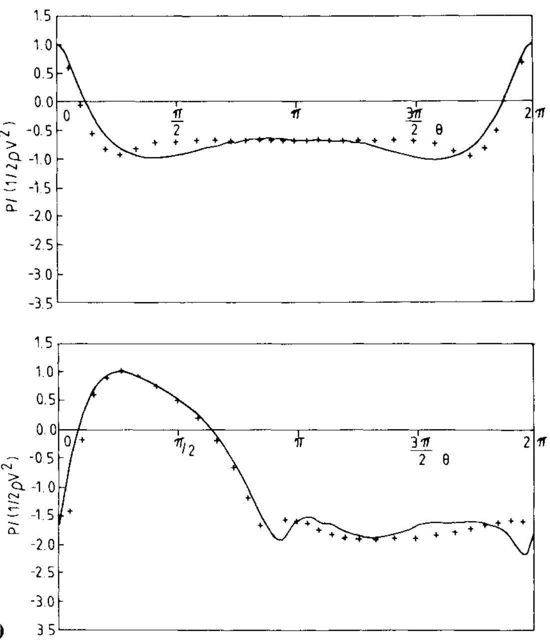

Fig. 1 **Mean pressure distributions about an elliptical cylinder of eccentricity 0.8 at two angles of attack: a) 0 deg, b) 60 deg. Θ is the angular position measured at the center of the ellipse with respect to the onset flow direction. The computer simulations are compared with experimental measurements. Experimental data,[5] + + +; computational results, —.**

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*Research Assistant, Department of Engineering, Simon Engineering Laboratories.

†Lecturer, Department of Engineering, Simon Engineering Laboratories.

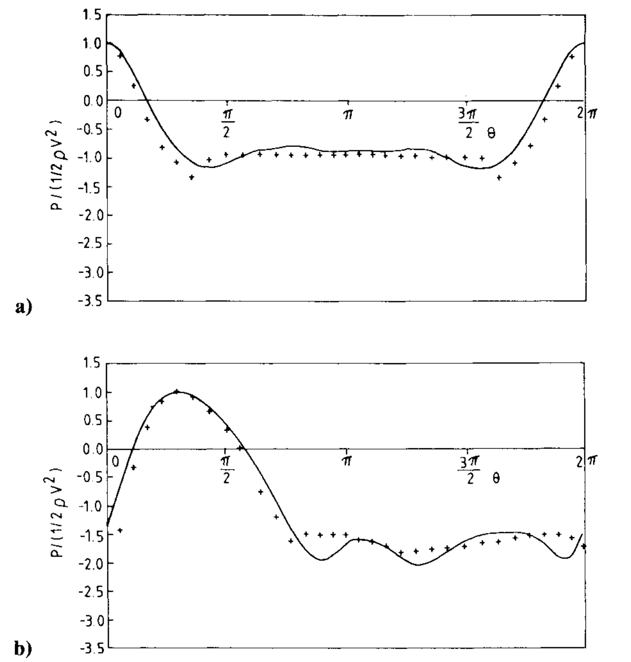

Fig. 2 Mean pressure distributions about an elliptical cylinder of eccentricity 0.6 at two angles of attack: a) 0 deg, b) 60 deg. The computer simulations are compared with experimental measurements. Experimental data,[5] + + +; computational results, —.

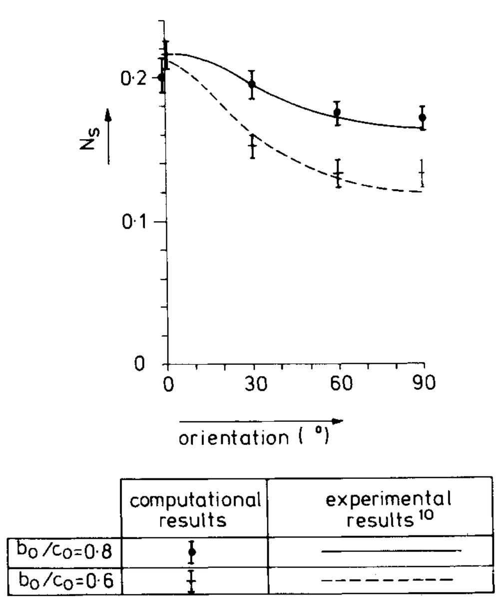

Fig. 3 A comparison with experimental measurement[5] of the computed variation of Strouhal number ($N_s = Fb/_0V$) with angle of attack (orientation) for two elliptical cylinders. F is the frequency of vortex shedding.

ρ is the fluid density. The pressure distribution around the surface of the cylinder is found by a summation of the pressure increments. Drag and lift coefficients are calculated from the pressure distribution.

Flow simulations were carried out for eight cylinder geometries for which experimental data are available.[5,6] The Reynolds number of the simulations was one to two orders of magnitude smaller than those of the experimental measurements. Pressures in the base region decrease as the Reynolds number is increased above 10^3 in contrast to experimental data for the circular cylinder.[7] This is attributed to an effective overprediction of the strength of the shed vortex clouds and is probably associated with the enforced two-dimensionality of the simulations, while experiments are known to contain three-dimensionality. The effect may be compensated for by introducing an empirical vortex decay law. Since, however, it is considered undesirable to introduce empirical factors, the simulations were performed for a Reynolds number of 10^3, based on the onset flow velocity and a typical length scale of each cylinder. The mean drag coefficients were calculated for a square cylinder, diamond cylinders of three fineness ratios, and two isosceles triangle cylinders (apex upstream and base upstream), each with rounded corners. Results are presented in Table 1. Mean pressure distributions and Strouhal numbers were calculated for elliptical cylinders of two eccentricities at various angles of attack. Results are presented in Figs. 1-3. Simulations up to a nondimensional time of 40, based on the onset flow velocity and a typical length scale of each cylinder, were possible with the computing resources available; this typically allowed the shedding of three or four vortex clouds. Time-averaging of the drag coefficiencies and pressure distributions was performed between times 20 and 40.

Table 1 Comparison of computed and experimental mean drag coefficients

Geometry	b_0/c_0	r_0/b_0	Drag coeff. (computed) $Vb_0/\upsilon = 10^3$	Drag coeff. (exp)[6] $Vb_0/\upsilon = 10^5$
Square	1.0	0.333	1.17	1.0
Diamond	2.0	0.167	1.85	1.7
	1.0	0.235	1.47	1.5
	0.5	0.333	1.07	1.1
Isosceles triangle				
Base forward	1.0	0.250	1.42	1.3
Apex forward	1.0	0.250	1.13	1.1

Note: b_0 and c_0 = frontal and streamwise dimensions of the cylinder, respectively; r_0 = corner radius; V = onset flow velocity.

Acknowledgments

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References

[1]Chorin, A. J., "Numerical Study of Slightly Viscous Flow," *Journal of Fluid Mechanics,* Vol. 57, March 1973, pp. 785-796.

[2]Stansby, P. K. and Dixon, A. G., "Simulation of Flows around Cylinders by a Lagrangian Vortex Scheme," *Applied Ocean Research,* Vol. 5, No. 3, 1983, pp. 167-178.

[3]Ives, D. C., "A Modern Look at Conformal Mapping Including Multiply Connected Regions," *AIAA Journal,* Vol. 14, Aug. 1976, pp. 1006-1011.

[4]Sudhoo, A., "Inviscid Compressible Flow past Multielement Aerofoils," Ph.D. Thesis, Univ. of Manchester, Manchester, England, 1985.

[5]Modi, V. J. and Wiland, E., "Unsteady Aerodynamics of Stationary Elliptic Cylinders in Subcritical Flow," *AIAA Journal,* Vol. 8, 1970, pp. 1814-1821.

[6]Delaney, N. K. and Sorensen, N. E., "Low-Speed Drag of Cylinders of Various Shapes," NACA TN-3038, Nov. 1953, pp. 1-22.

[7]Smith, P. A., "Computation of Viscous Flows by the Vortex Method," Ph.D. Thesis, Univ. of Manchester, Manchester, England, 1986.